Workshop “Geometry, Probability and Topology in Applications”

September 27 – October 1, 2025

Belgrade, Serbia

BOOK OF ABSTRACTS

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The aim of our workshop

The "Geometry, Probability and Topology in Applications" (GPTA 2025) workshop aims to foster scientific exchange and encourage collaboration between researchers working in geometry, topology, probability, and related applied fields. The primary goal of the workshop is to initiate new research directions and strengthen international academic networks through discussions and knowledge sharing. As a valuable additional outcome, the workshop also provides a platform for young researchers from Serbia to present their work and become more visible within the broader scientific community, in accordance with the bylaw of the Serbian Ministry of Science, Technological Development and Innovation.

Program Overview

The two-day workshop features a series of lectures from researchers across Europe and beyond, covering both theoretical advances and applications. The event includes coffee breaks and open problems session in order to encourage collaboration. The program of the workshop should provide an opportunity for informal exchange of ideas and networking.

Acknowledgements

We express our sincere gratitude to Academician Stevan Pilipović for his support, and to Professor Zoran Ognjanović, Director of the Mathematical Institute SANU, for his contribution to the organization of this event. We also thank the Tourist Office of Serbia and Agency Impala for their generous assistance.

Blurred Magnitude Homology of Functional Connectome for ASD Detection

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Autism spectrum disorder (ASD) is one of the most common neurodevelopmental disorders. Existing studies show that adults with ASD may experience accelerated or altered neurocognitive aging. Autism alters functional connectivity between brain regions. Therefore, it is important to develop methods for diagnosing this condition based on the analysis of brain network. Functional brain networks are usually studied using undirected correlations, while functional connections in the brain are inherently directed. Blurred magnitude homology is an algebro-topological tool that allows analyzing directed graphs including directed functional connectomes. The method proposed in this work is based on applying a fully connected neural network to blurred magnitude homology-based features of a directed brain network of functional connectivity. Experiments performed on real connectomes constructed from fMRI images show that blurred magnitude homology is a useful invariant for distinguishing directed brain networks of individuals with ASD and typically developing individuals.

This is joint work with Alexandr Kachura.

Geometries That Mapper: Topology in Data and Materials

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Topological Data Analysis (TDA) is a powerful framework for exploring the shape of complex, high-dimensional data. In this talk, I will present both classical and modern applications of TDA, highlighting its versatility across scientific domains. I will begin with Mapper-based visualizations, which have provided new insights in biomedical and chemical data-ranging from the classification of breast cancer subtypes, through analysis of wine samples from spectroscopy, to the structural organization of tissues. I will then turn to persistent homology and its growing role in material science. Specifically, I will demonstrate how TDA enables the characterization of phase separation in alloys, supports the exploration of the vast design space of zeolites, and assists in identifying materials with optimal pore geometry and maximal resistivity. These examples illustrate how topological techniques not only provide qualitative visualization, but also serve as quantitative tools guiding material discovery and optimization.

The finite groups of random walks in the quarter plane

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We consider maximally space homogeneous random walks as a class of discrete time homogeneous Markov chains, with the state space being the quarter plane. The generators of the process in this region are $\{p_{ij}| -1 \leq i, j \leq 1\}$, where p_{ij} is the transition probability for the jump from (r, s) to $(r + i, s + j)$, for $rs > 0$. This leads to the consideration of a biquadratic curve $C(P)$ in the plane $P^1 \times P^1$, given by its affine equation in $\mathbb{C}^2$:

$$C(P)_A : Q_P(x, y) = xy \left(\sum_{i,j=-1}^{1} p_{ij} x^i y^j - 1 \right) = 0, p_{ij} \geq 0, \sum_{i,j=-1}^{1} p_{ij} = 1.$$

There is the vertical and horizontal switch, v and h defined on the biquadratic curve $C(P)$. The group of random walk in the quarter plane $H(P)$ is isomorphic to the group of automorphisms of the curve, generated with these two switches:

$$H(P) :=< h, p | h^2 = Id, p^2 = Id >,$$

We describe the situations with finite groups of random walks. This is a work in progress, joint with M. Radnovic.

The expected length of a Euclidean minimum spanning tree and the expected 1-norms of chromatic persistence diagrams

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A classic result on Euclidean minimum spanning trees (EMST) is the existence of an asymptotic constant, c, such that the expected length of the EMST of n points sampled uniformly at random in the unit square is c times the square root of n. However, the value of c is not known. Prior to this work, the known bounds were $0.6008 < c < 0.7072$, and we improve the lower bound to $0.6289 < c$.

The motivation for this work is the stochastic analysis of chromatic persistence diagrams. In particular, we show that similar asymptotic constants exist for the expected 1-norms of all diagrams in the 6-pack of a randomly 2-colored set of points in the unit square, in which we study the inclusion of the disjoint union of the sublevel sets of the two monochromatic distance functions into the bichromatic distance function.

This is joint work with Ondrej Draganov, Sophie Rosenmeier, and Morteza Saghafian.

Fundamental groups of small simplicial complexes

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Minimal triangulations of manifolds have been widely studied, but many problems remain open. I will present a computation of the fundamental groups of simplicial complexes with at most 8 vertices and explain how it can be used to obtain some new estimates for the number of vertices in minimal triangulations of manifolds. I will also explain how these results can be obtained in a purely geometric manner for 6 and 7 vertices and discuss some partial results for the 9 vertex case.

LLN for the bigraded Betti numbers of the random moment-angle complex

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My plan is to carefully recall the framework and the main result of a joint work with Djordje Baralić. The random moment-angle complex (RMAC) is a natural generalization of the Erdõs-Rényi-Stepanov random graph. RMAC is a model from probabilistic algebraic topology. Its definition, like the study of its properties, rely on the synergy of algebraic topology and probability theory.

Topological Data Analysis of Classical and Quantum Invariants

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Topological Data Analysis (TDA) provides a rigorous framework for extracting structural insights from complex data by probing its underlying shape. In this talk, I present recent advances in visualizing and analyzing maps between high-dimensional spaces, motivated by challenges in knot theory. The exponential growth in knot complexity positions the space of knots and their invariants as a compelling case study in mathematical "big data". We focus on the Alexander and Jones polynomials, as well as Khovanov link homology, highlighting applications that recover classical results, reveal new patterns, and even disprove conjectures. These methods illustrate how TDA can be used in data rich areas of theoretical mathematics, providing conceptual understanding and practical tools for formulating new and addressing open conjectures.

On symplectic reductions for the canonical T^n-action on $G_{n,2}$

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The complex Grassmann manifolds $G_{n,2}$ are of specific mathematical interest, since, among the other, from the canonical actions of an algebraic torus $(\mathbb{C}^*)^n$ and the compact torus T^n on these manifolds many structures arise, which are closely related to many important problems in different areas of mathematics. One of such structures is a symplectic reduction.

The canonical T^n-action on $G_{n,2}$ is Hamiltonian and the induced moment map is the standard one $\mu : G_{n,2} \to \Delta_{n,2}$ for the hypersimplex $\Delta_{n,2} \subset \mathbb{R}^n$, which is defined by the Plücker equivariant embedding $G_{n,2} \subset \mathbb{C}P^N$, $N = \binom{n}{2} - 1$. The symplectic manifolds $\mu^{-1}(x)/T^n$, known as symplectic reductions, arise for the regular values x, defined in the classical sense, of the moment map μ.

We discuss [3] the problem of description of smooth manifolds $\mu^{-1}(x)$ for the Grassmannians $G_{n,2}$ which produce symplectic reductions. In this context, we explain in detail the Grassmann manifold $G_{4,2}$ and prove that $\mu^{-1}(x) \cong S^3 \times T^2$ for any regular value $x \in \Delta_{4,2}$.

In addition, we show that the Deligne-Mumford compactifications $\overline{\mathcal{M}}_{0,n}$ and Losev-Manin compactifications $\bar{L}_{0,n}$ may be realized as symplectic reductions for the canonical T^n-action on $G_{n,2}$ if and only if $n = 4, 5$.

The talk is based on the joint works with Victor M. Buchstaber.

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Barcoded Characteristic Closeness of Motion Vector Field Stability Persistence Recorded in Sequences of Video Frames

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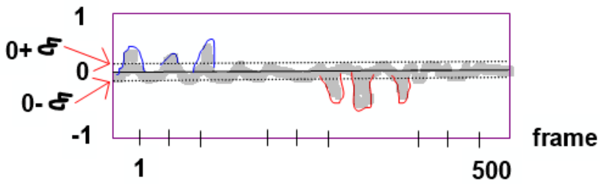

Figure 1: Barcode Stability Persistence

This paper introduces the geometry of persistence of characteristically near video motion vector fields via barcodes. Let $\mathbf{V}f_{frA}, \mathbf{V}f_{frB} \in 2^{\mathbb{C}}$ be video frame motion vector fields in the polar complex plane containing vectors $\mathbf{v}_{frA}, \mathbf{v}_{frB}$ and let $\varphi(\mathbf{v}_{frA}), \varphi(\mathbf{v}_{frB})$ be motion vector characteristics. The characteristic distance mapping [1] $d^{\Phi} : 2^X \times 2^Y \to \mathbb{C}$ is defined by

$$\boxed{d^{\Phi}(\mathbf{V}f_{frA}, \mathbf{V}f_{frB}) = \inf |\varphi(\mathbf{v}_{frA}) - \varphi(\mathbf{v}_{frB})| = \varepsilon \in [-1, 1] \in \mathbb{C}.}$$

Here, we use barcodes and persistence homology [2] instead of the usual homology class language of birth, persistence, death that is replaced by beginning, $\varepsilon \in [-1, 0)$ (failure), $\varepsilon \in [0, 1)$ (fractional vector field stability adherence) intervals, end of evolutionary interval. A barcode provides a simple descriptor for topological evolution of motion vector field stability recorded in sequences of video frames (see, e.g., Fig. 1). Stability is measured in terms of maximal eigenvalues using the Krantz criterion [3] ($\lambda_{max} \in [0, 1] \in \mathbb{C}$ indicates field stability). The shorter the temporal interval, the more likely the characteristic closeness of stable motion vector fields.

The talk is based on the joint works with James F. Peters.

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Polynomial entropy on the n-fold symmetric product and its suspension

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Let $f\colon X \to X$ be a homeomorphism on a compact metric space X. We are interested in investigating the complexity of induced dynamical systems on hyperspaces. The hyperspace $F_n(X)$ of all nonempty subsets with at most n points (for all $n \in \mathbb{N}$), will be of main interest in our talk, as well as the n-fold symmetric product suspension $\mathcal{S}F_n(X) = F_n(X)/F_1(X)$, $n \geq 2$.

Tetraplectic structures compatible with local quaternionic toric structures

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In this talk, we discuss a quaternionic analogue of toric geometry via tetraplectic structures compatible with local quaternionic torus actions on $4n$-manifolds. Using local models of $\mathrm{Sp}(1)^n$, we define quaternionic Lagrangian-type fibrations and prove a quaternionic Arnold-Liouville theorem. Orbit spaces inherit a quaternionic affine structure, classified by a Delzant-type correspondence.

This is joint work with P. Batakidis (arXiv:2506.10148).

Vertex degrees in β- and β'-Delaunay graphs

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The β-Delaunay and β'-Delaunay triangulations are tessellation models generalizing classical Poisson-Delaunay triangulation on the Euclidean space. Both were introduced in recent years by Gusakova, Kabluch–ko, and Thäle in a series of papers. We present various results on the degrees of the vertex-edge graphs of these models, including bounds on the tail distributions of typical degrees and (in one of those models) a concentration phenomenon for the maximal degree in a growing window.

Sperner's colorings of hypergraphs arising from edgewise triangulations

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Motivated by the work of Mirzakhani and Vondrák, we investigate combinatorial properties of the edgewise triangulation of a simplex. For a given permutation π, the facets of this triangulation define the hyperedges of a hypergraph H^{π}. The optimal Sperner coloring of H^{π} is determined by G_{π}, the graph associated with π. We also study a related simplicial complex naturally arising from π.

Two principles of pseudo-Riemannian Osserman tensors and manifolds

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We introduce the duality and orthogonality principles of Osserman tensors as they are properties of curvature tensors that are characteristic of Riemannian Osserman manifolds. It is known that Riemannian algebraic curvature tensor is Osserman if and only if it is Jacobi-dual. Every Riemannian Jacobi-orthogonal tensor is an Osserman tensor, while Clifford and two-root Riemannian Oserman tensors are Jacobi-orthogonal. We generalize the established principles to the pseudo-Riemannian case.

An Explicit Formula for Vertex Enumeration in the $\mathrm{CUT}(n)$ Polytope via Probabilistic Methods

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Building on a probabilistic interpretation via agreement probabilities among symmetric Bernoulli random variables, we derive an explicit closed-form formula for enumerating the vertices of the $\mathrm{CUT}(n)$ polytope. The approach uses a natural binary encoding and a novel integer generating map rich in symmetries and self-similarity. It is the first explicit vertex enumeration formula for this classical polytope family.